Homework 9

Mathematical foundations of informatics (I201, 2008)

Instructor: Tang

(This HW will be collected on 12/3 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. **NO late homework will be accepted. Problem 5 is optional.**)

1. (16 pts) Consider a first order language that consists of one binary predicate symbol R. Let Ψ be the formula: $\exists x \exists y \forall z((x \neq y) \wedge (R(x,z) \leftrightarrow R(y,z)))$.
   a. Let U={a, b, c} and I(R)={(c,b),(b,b),(a,b)}, is Ψ valid in this model M={U, I}? Justify your answer.
   b. Let U'={a, b, c} and I'(R)={(c,b), (b,b), (b,a)}, is Ψ valid in this model M={U', I'}? Justify your answer.

2. (18 pts) Consider a first order language that consists of one binary predicate T and one unary predicate W. Assume the model M = {U, I}, where U = {a, b, c}, I(T) = {(a, a), (a, b), (b, b)}, I(W) = {b, c}. Tell whether each of the following formulas is valid under the model M.
   a. $\forall x(\exists y T(x, y) \wedge W(x))$
   b. $\forall x(W(x) \rightarrow \forall y T(x, y))$
   c. $\exists x \exists y((x \neq y) \wedge W(x) \wedge \neg T(x, y))$

3. (16 pts) Consider a first order language that consists of one binary predicate symbol R and one unary predicate symbol T. Assuming U={a, b, c, d}, for each of the following formulas, find an interpretation I to make it true, and another interpretation I' to make it false. Be precise in your reasoning and explain your answer. Note: you need to define both I(T) and I(R) for I.
   a. $\exists x(T(x) \wedge \forall y R(x,y))$
   b. $\forall x \exists y(T(x) \rightarrow \neg R(x,y))$

4. (10 extra pts) Show the arguments below are universally valid by using set theory.
   a. $\exists x P(x) \rightarrow \forall x Q(x)$
      $P(a)$
      ____
      $Q(a)$

   b. $\forall x(\neg P(x) \vee Q(x))$
      $P(a)$
      ____
      $Q(a)$