Homework 7
Mathematical foundations of informatics (I201, 2008)
Instructor: Tang

(This HW will be collected on 11/10 Mon. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. **NO late homework will be accepted.**)

1. (8 pts) For each of the relations below, tell whether it is reflexive, symmetric, transitive, anti-symmetric, and/or equivalence relation.
    a. Let R be the relation on $\mathbf{Z}$, where R = {(a, b)| a < b};
    b. Let R be the relation on $\mathbf{Z}$, where R = {(a, b)| a ≤ b};
    c. Let A = {x | x is a book}, and let R be the relation on A, where R = {(a, b)| a and b are published by the same publisher.};
    d. Let A be the set of all cities. Define the relation R on A to be R = {(a, b)| you can drive on roads from a to b.}

2. (6 pts) Let L be a binary predicate symbol, and F be a binary function. Which of the following expressions are *terms* in predicate logic? Which of the following expressions are predicate logic *formulas*?
    a. ∃x∀y L(x,y)
    b. ∃x∀y L(x,y) ∧∃xL(x, F(a,b))
    c. ∃x∃y F(x,y)
    d. L(a,b) → F(x,y)
    e. F(a,F(a,b))
    f. (a,b)

3. (4 pts) Show that the formula $\forall x A(x) \rightarrow \exists x A(x)$ is universally valid.

4. (12 pts) Let K be a unary predicate symbol, and H be a binary predicate symbol. K(x) means "x watched the TV show last night"; H(x, y) means "x talks about the TV show with y". Assume the universe of discourse is all the students in the class. Translate the following sentences to predicate logic formula.
    a. Nobody in the class watched the TV show last night.
    b. There is at least one student in the class who watched the TV show last night.
    c. Every student who watched the TV show last night talked with somebody else about it.
    d. Two students who watched the TV show talked with each other about it.
    e. One student who watched the TV show last night talked about it with another student who did not watch the TV show last night.
    f. Not every student who watched the TV show last night talked with somebody in the class about it.

5. (12 pts) Let K be a unary predicate symbol, and H be a binary predicate symbol. K(x) means "x watched the TV show last night"; H(x, y) means "x talks about the TV show with y". Assume the universe of discourse is all the students in the class. Translate the following formulas into English sentences (clear and grammatically correct English!).

a. $\exists x \forall y ((x \neq y) \wedge K(x) \rightarrow H(x,y))$
b. $\neg(\forall x(K(x)))$
c. $\forall x \forall y ((x \neq y) \wedge K(x) \wedge K(y) \rightarrow \neg H(x,y))$
d. $\exists x \exists y \forall z ((x \neq y) \wedge (H(x,z) \leftrightarrow H(y,z)))$

6. (4 pts) Consider a first order language that consists of one binary predicate symbol R. Let Ψ be the formula: $\exists x \exists y \forall z ((x \neq y) \wedge (R(x,z) \leftrightarrow R(y,z)))$.

    a. Let U={a, b, c, d} and I(R)={(c,b),(b,b),(a,b)}, is Ψ valid in this model M={U, I}? Justify your answer.

    b. Let U'={a, b, c, d} and I'(R)={(c,b),(b,b),(b,a)}, is Ψ valid in this model M={U', I'}? Justify your answer.

7. (4 pts) Consider a first order language that consists of one binary predicate symbol R and one unary predicate symbol T. Assuming U={a, b, c, d}, for each of the following formula, find an interpretation I to make it true, and another interpretation I' to make it false. Be precise in your reasoning and explain your answer. Note: you need to define both I(T) and I(R) for I.

    a. $\exists x (T(x) \wedge \forall y R(x,y)) \wedge \exists y (T(y) \wedge \forall x R(x,y))$
    b. $\forall x \exists y (T(x) \rightarrow \neg R(x,y))$