Homework 6
Mathematical foundations of informatics (I201, 2008)
Instructor: Tang

(This HW will be collected on 10/29 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. **NO late homework will be accepted.**)

1. (3 pts) Use set builder notation to give a description of each of these sets. That is you need to find a property that describes the elements in the set.
    a. {0, 3, 6, 9, 12}
    b. {−3,−2,−1, 0, 1, 2, 3}
    c. {1, 3, 5, 7, · · · }

2. (8 pts) Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. Find
    a. $A \cap B$
    b. $A \cup B$
    c. $A - B$
    d. $B - A$

3. (3pts) Suppose that A = {2, 4, 6}, B = {2, 6}, C = {4, 6}, and D = {4, 6, 8}. Determine which of these sets are subsets of which other of these sets.

4. (3 pts) Suppose that A = {2, 5, 7}, B = {5, {7}, 2}, C = {5, 7, 4}. Draw the Venn diagram to represent the relationships.

5. (7pts) Determine whether each of these statements is true or false:
    a. $0 \in \varnothing$;
    b. $\{0\} \in \varnothing$;
    c. $\{0\} \subseteq \varnothing$;
    d. $0 \in \{0\}$;
    e. $\{0\} \subseteq \{0\}$;
    f. $\varnothing \subseteq \{0\}$;
    g. $\varnothing \in \{0\}$.

6. (3pts) Determine whether each of these sets is the power set of a set.
    a. $\varnothing$;
    b. $\{\varnothing, \{a\}\}$;
    c. $\{\varnothing, \{a\}, \{\varnothing, \{a\}\}\}$.

7. (3 pts) Suppose that A × B = $\varnothing$, where A and B are two sets. What can you conclude? Explain.

8. (6 pts) What is the cardinality of each of these sets? Note $\mathcal{P}(A)$ is the power set of the set A.
    a. $\varnothing$;
    b. $\{\varnothing\}$;
    c. $\{\varnothing, \{\varnothing\}, \{\varnothing, a\}\}$;

d. $\mathcal{P}(\{a, b, \{a, b\}\})$

e. $\mathcal{P}(\mathcal{P}(\varnothing))$

f. $\mathcal{P}(\{\varnothing\})$

9. (4 pts) Let A = {a, b, c}, B = {x, y} and C = {0, 1}. Find
   a. $A \times B \times C$;
   b. $C \times B \times A$;
   c. $C \times A \times B$;
   d. $B \times B \times B$.

10. (4 pts) Let A = {a,b,c,d,e}, and define two relations on A as
$$R_1 = \{<a,a>,<a,b>,<b,a>,<b,b>,<c,c>,<d,d>,<d,e>,<e,d>,<e,e>\};$$
$$R_2 = \{<a,b>, <b,a>, <b,b>, <c,c>, <d,d>, <d,e>\}$$
Is R_1 an equivalence relation? Is R_2 an equivalence relation? Explain your answers.

11. (6 pts) For each of the following cases, determine if **f** is a function. If it is a function, determine if it is an injective (one-to-one) function, a surjective (onto) function and bijective function. Explain your answers.
   a. A={1,2,3,4,5}, B={6,7,8,9,10}, f = {<1,8>, <3,9>, <4,10>, <2,6>, <5.9>};
   b. A={1,2,3,4,5}, B={6,7,8,9,10}, f = {1,8>, <3,10>, <2,6>, <4,9>}
   c. A={1,2,3,4,5}, B={6,7,8,9,10}, f = {<1,7>, <2.6>, <4.5>, <1,9>, <5,10>}.