

## Homework 4

Mathematical foundations of informatics (I201, 2008)

Instructor: Tang

(This HW will be collected on 10/1 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. **NO late homework will be accepted.** )

1. (18 pts) Check the following arguments for validity. In each case, either prove that it is valid or show by a counterexample that it is invalid.

a.  $B \rightarrow \neg S \vee M$

$$S \vee M$$

$$M \rightarrow H$$

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$$B \rightarrow H$$

b.  $A$

$$A \rightarrow B$$

$$B \rightarrow C$$

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$$C$$

c.  $A \rightarrow (B \wedge C)$

$$\neg B \rightarrow \neg A$$

$$\neg C$$

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$$A \rightarrow (B \vee C)$$

2. (10 pts) Are the following statements consistent? If yes, give truth values for the atomic propositions involved that make all statements true; otherwise, give a proof.

a. The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

b. Users cannot access the file system whenever the system software is being upgraded. Users can access the file only if they can save new files. If users can save new files, then the system software is not being upgraded.

3. (10pts) Are the following arguments valid? If yes, give a proof; otherwise, give a counterexample.

a. The Republicans will lose power only if the dollar drops or the debt increases. If the dollar drops and the debt increases, then the Republicans will lose power. The debt does not increase if the dollar does not drop. The dollar drops. Therefore the Republicans will lose power.

b. The law is unnecessary, given that people are good. As long as people are not good, the law is ineffective. So, either the law is unnecessary or ineffective.

4. (12 pts) There is an island in Pacific called the Island of Knights and Knaves. There are two groups of inhabitants on this island, namely Knights who always tell the truth and Knaves who always lie. Can you solve the following puzzles:
- a. We have three inhabitants A, B, and C on the Island of Knights and Knaves. Suppose A and B say the following:
    - A: All of us are knaves.
    - B: Exactly one of us is a knave.Can you determine what C is? Explain your answer.
  - b. We have three inhabitants A, B, and C on the Island of Knights and Knaves. A and B make the following statements:
    - A: "I am a knave but B is not a knave."
    - B: "All of us are knights."Can you determine what A, B and C are? Explain your answer.