Homework 3
Mathematical foundations of informatics (I201, 2008)
Instructor: Haixu Tang

(This HW will be collected on 9/24 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. NO late homework will be accepted. Note: have plenty of scratch sheets of paper available and liberally use them.)

1. Check if the formulas below are tautologies using truth trees. Recall that in case the formula is not a tautology you need to provide a counterexample that makes the formula false.
   a. \( A \leftrightarrow (B \rightarrow A \land B) \)
   b. \( (A \land (B \rightarrow C)) \land (B \land \neg C) \)
   c. \( (D \lor B) \rightarrow (D \land B) \)
   d. \( (D \land B) \rightarrow (D \lor B) \)
   e. \( ((A \lor B) \land \neg A) \rightarrow B \)

2. Give an example of a formula that is not a tautology whose truth tree has all open branches. Include both the formula and its truth tree in your answer.

3. Show that \( \neg P \rightarrow (Q \rightarrow R) \) and \( Q \rightarrow (P \lor R) \) are logically equivalent using truth trees.

4. Show that \( A \land (B \rightarrow C) \equiv \neg (A \rightarrow B) \lor (A \land C) \), using logical laws.

5. Show that \( (\neg A \rightarrow B) \leftrightarrow (A \lor B) \) is a tautology, using logical laws.

6. Show that \( (P \lor Q) \rightarrow R \equiv (P \rightarrow R) \land (Q \rightarrow R) \), using logical laws.

7. Are the following sets of formulas consistent? If the set is not consistent, use the truth tree to disprove it; if the set is consistent, given an assignment of truth values.
   a. \( \{A, A \rightarrow B, B \lor C, D\} \)
   b. \( \{A \land B, B \rightarrow C, \neg C \land A\} \)
   c. \( \{A \rightarrow B, B \rightarrow C, A, \neg C\} \)