Homework 3

Mathematical foundations of informatics (I201, 2008)

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(This HW will be collected on 9/24 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. **NO late homework will be accepted. Note: have plenty of scratch sheets of paper available and liberally use them.)**

1. Check if the formulas below are tautologies using truth trees. Recall that in case the formula is not a tautology you need to provide a counterexample that makes the formula false.
   a. $A \leftrightarrow (B \rightarrow A \wedge B)$
   b. $(A \wedge (B \rightarrow C)) \wedge (B \wedge \neg C)$
   c. $(D \vee B) \rightarrow (D \wedge B)$
   d. $(D \wedge B) \rightarrow (D \vee B)$
   e. $((A \vee B) \wedge \neg A) \rightarrow B$

2. Give an example of a formula that is not a tautology whose truth tree has all open branches. Include both the formula and its truth tree in your answer.

3. Show that $\neg P \rightarrow (Q \rightarrow R)$ and $Q \rightarrow (P \vee R)$ are logically equivalent using truth trees.

4. Show that $A \wedge (B \rightarrow C) \equiv \neg(A \rightarrow B) \vee (A \wedge C)$, using logical laws.

5. Show that $(\neg A \rightarrow B) \leftrightarrow (A \vee B)$ is a tautology, using logical laws.

6. Show that $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$, using logical laws.

7. Are the following sets of formulas consistent? If the set is not consistent, use the truth tree to disprove it; if the set is consistent, given an assignment of truth values.
   a. $\{A, A \rightarrow B, B \vee C, D\}$
   b. $\{A \wedge B, B \rightarrow C, \neg C \wedge A\}$
   c. $\{A \rightarrow B, B \rightarrow C, A, \neg C\}$