Homework 2

Mathematical foundations of informatics (I201, 2008)

(This HW will be collected on 9/17 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. **NO late homework will be accepted.**)

1. (9 pts) Construct the truth table for each of the following formulas:
   a. $Z \rightarrow Z \wedge A$
   b. $A \vee A \leftrightarrow C$
   c. $\text{Pumpkin} \rightarrow \neg\neg(A \leftrightarrow \text{Pumpkin})$
2. (4 pts) We'll define a new connective "□". The meaning of (A □ B) is represented by the following truth table:

A	B	A □ B
t	t	t
t	f	t
f	t	f
f	f	t

Construct the truth table for the formula $(\neg A \leftrightarrow B) \square (A \leftrightarrow \neg B)$.

3. (9 pts) In class we saw that there's a way to establish whether two propositions A and B are logically equivalent (or semantically identical–that they have the same meaning). Here are the steps: (1) create a truth table for $A \leftrightarrow B$; and (2) check whether it's a tautology. If it's a tautology, then A and B mean the same thing. Use this procedure to check whether the following pairs of formulae are identical:
   a. $A \rightarrow B$ and $\neg A \vee B$
   b. Zebra $\wedge \neg$Zebra and $\neg$ (Zebra $\vee \neg$Zebra)
   c. Bee $\rightarrow \neg$ flower and flower $\rightarrow$ Bee

4. (8pts) Is a tautology a contingency? Is a tautology satisfiable? Is a satisfiable formula a tautology? Can a satisfiable formula be a contradiction? Explain your answers.

5. (8 pts) Which of the following are tautologies? Use truth table to prove or provide a counterexample to disprove.
   a. $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$
   b. $(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$
   c. $\neg (A \wedge B) \leftrightarrow (\neg A \vee \neg B)$
   d. $A \rightarrow B \leftrightarrow \neg A \rightarrow \neg B$

6. (12 pts) Use truth trees to check which of the following formulas are satisfiable? In case the formula is satisfiable, provide an example that makes the formula true.
   a. $A \rightarrow (A \wedge B)$
   b. $A \wedge (B \rightarrow C) \wedge (B \wedge \neg C)$
   c. $A \wedge B \rightarrow A \vee B$
   d. $A \vee B \rightarrow A \wedge B$