Homework 2
Mathematical foundations of informatics (I201, 2008)

(This HW will be collected on 9/17 Wed. in the class. Write LEGIBLY and explain your answers clearly. The homework you hand in must be your own work, IN YOUR OWN WORDS and your own explanation. NO late homework will be accepted.)

1. (9 pts) Construct the truth table for each of the following formulas:
   a. \( Z \rightarrow Z \land A \)
   b. \( A \lor A \leftrightarrow C \)
   c. Pumpkin \( \rightarrow \neg (A \leftrightarrow \text{Pumpkin}) \)

2. (4 pts) We’ll define a new connective “□”. The meaning of \( (A \square B) \) is represented by the following truth table:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A \square B</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>t</td>
<td>t</td>
</tr>
<tr>
<td>t</td>
<td>f</td>
<td>t</td>
</tr>
<tr>
<td>f</td>
<td>t</td>
<td>f</td>
</tr>
<tr>
<td>f</td>
<td>f</td>
<td>t</td>
</tr>
</tbody>
</table>

Construct the truth table for the formula \( (\neg A \leftrightarrow B) \square (A \leftrightarrow \neg B) \).

3. (9 pts) In class we saw that there’s a way to establish whether two propositions \( A \) and \( B \) are logically equivalent (or semantically identical–that they have the same meaning). Here are the steps: (1) create a truth table for \( A \leftrightarrow B \); and (2) check whether it’s a tautology. If it’s a tautology, then \( A \) and \( B \) mean the same thing. Use this procedure to check whether the following pairs of formulae are identical:
   a. \( A \rightarrow B \) and \( \neg A \leftrightarrow B \)
   b. \( \text{Zebra} \land \neg \text{Zebra} \) and \( \neg (\text{Zebra} \lor \neg \text{Zebra}) \)
   c. \( \text{Bee} \rightarrow \neg \text{flower} \) and \( \text{flower} \rightarrow \text{Bee} \)


5. (8 pts) Which of the following are tautologies? Use truth table to prove or provide a counterexample to disprove.
   a. \( (A \rightarrow B) \land (B \rightarrow C) \rightarrow (A \rightarrow C) \)
   b. \( (A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A) \)
   c. \( \neg (A \land B) \leftrightarrow (\neg A \lor \neg B) \)
   d. \( A \rightarrow B \leftrightarrow \neg A \rightarrow \neg B \)

6. (12 pts) Use truth trees to check which of the following formulas are satisfiable? In case the formula is satisfiable, provide an example that makes the formula true.
   a. \( A \rightarrow (A \land B) \)
   b. \( A \land (B \rightarrow C) \land (B \land \neg C) \)
   c. \( A \land B \rightarrow A \lor B \)
   d. \( A \lor B \rightarrow A \land B \)